Nonlinear evolution of pressure gradient driven modes and anomalous transport in plasmas

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outline

1. Formulation: MHD
2. Rayleigh-Taylor instability
3. Rayleigh-Bénard convection
4. Resistive interchange mode
5. Nonlinear analyses and bifurcation
6. tearing modes of multiple rational surfaces
7. Nonlinear evolution of multiple tearing modes
8. Summary

MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{v}) = 0$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{v} \right) = -\nabla p + \mathbf{j} \times \mathbf{B} + \nu \Delta \mathbf{v}$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \bullet \nabla p + \gamma p \nabla \bullet \mathbf{v} = \kappa \Delta p$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{j})$$

$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B} \qquad \nabla \bullet \mathbf{B} = 0$$

Fluid dynamics equations

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{v}) = 0$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{v} \right) = -\nabla p - \rho g \mathbf{z} + \nu \Delta \mathbf{v}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \bullet \nabla T + (\gamma - 1) \gamma T \nabla \bullet \mathbf{v} = \kappa \Delta T$$

$$p = p(\rho, T)$$

with the gravitational field in the negative z direction

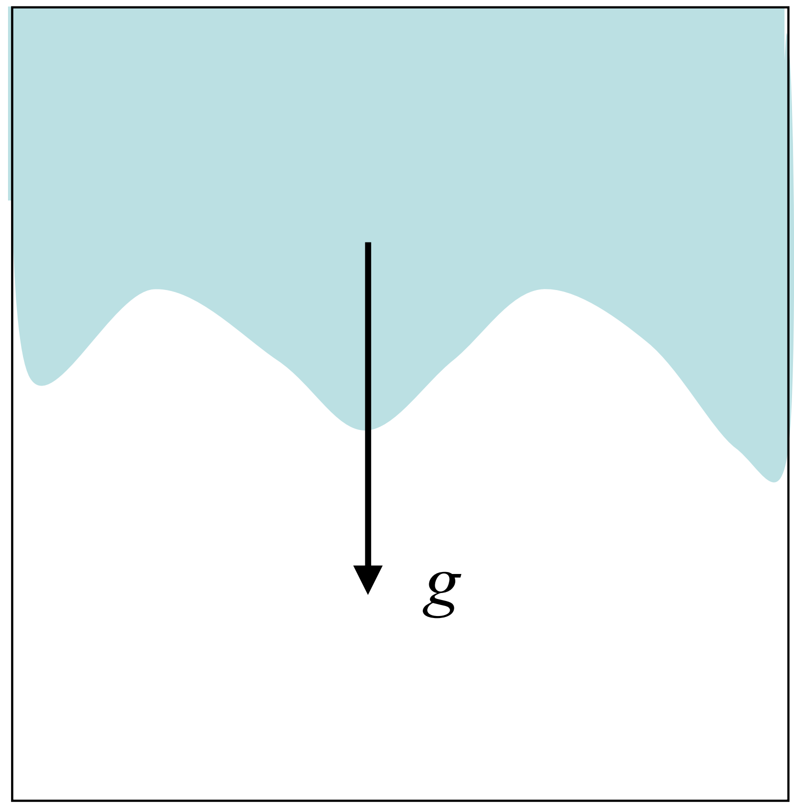
Rayleigh-Taylor instability

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \bullet \nabla \rho = 0$$

$$\nabla \bullet \mathbf{v} = 0$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{v} \right) = -\nabla p - \rho g \mathbf{z}$$

incompressibility is assumed



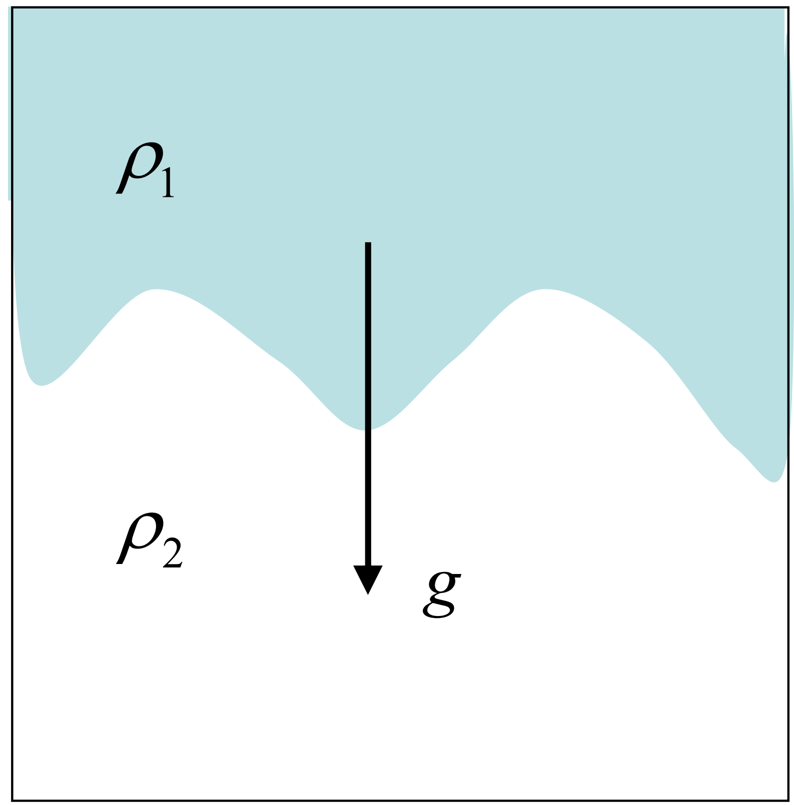
Rayleigh-Taylor instability

often analyzed for two separate fluids

$$\rho = \rho_1 \quad \text{or} \quad \rho_2$$

$$\nabla \bullet \mathbf{v} = 0$$

$$\rho_i \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{v} \right) = -\nabla p - \rho_i g \mathbf{z}$$



Rayleigh Bénard convection

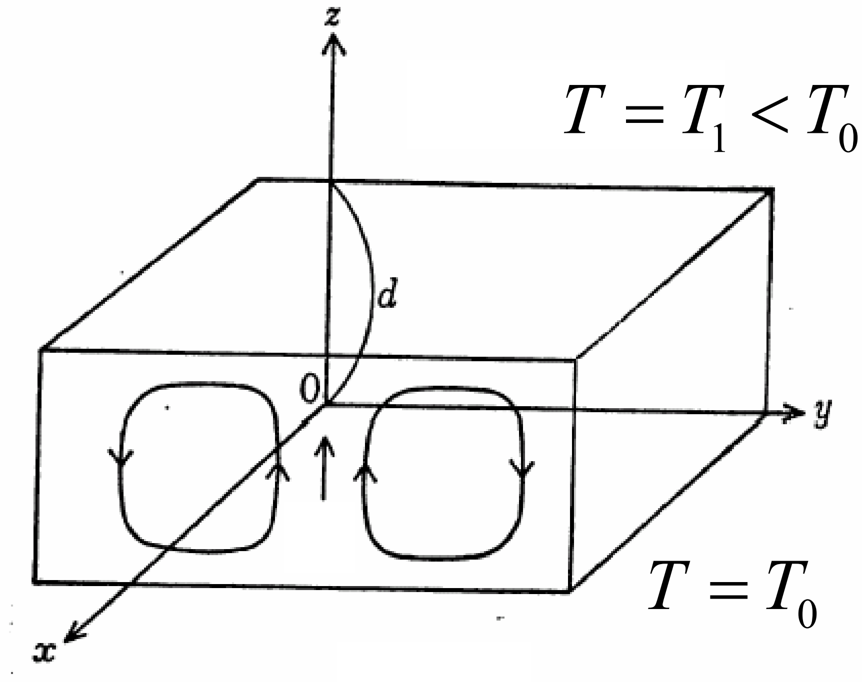
$$\nabla \bullet \mathbf{v} = 0$$

$$\rho_0 \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{v} \right) = -\nabla p - \rho g \hat{\mathbf{z}} + \nu \Delta \mathbf{v}$$

$$\rho(T) = \rho_0 \left[1 - \alpha (T - T_0) \right]$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \bullet \nabla T = \kappa \Delta T$$

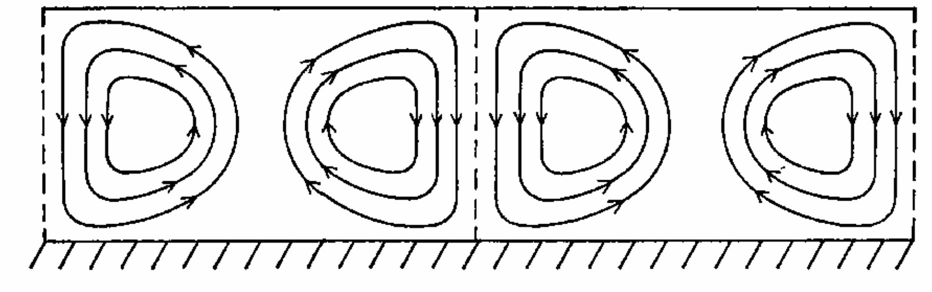
Boussinesq approximation



Rayleigh Bénard convection

2D convection : rolls

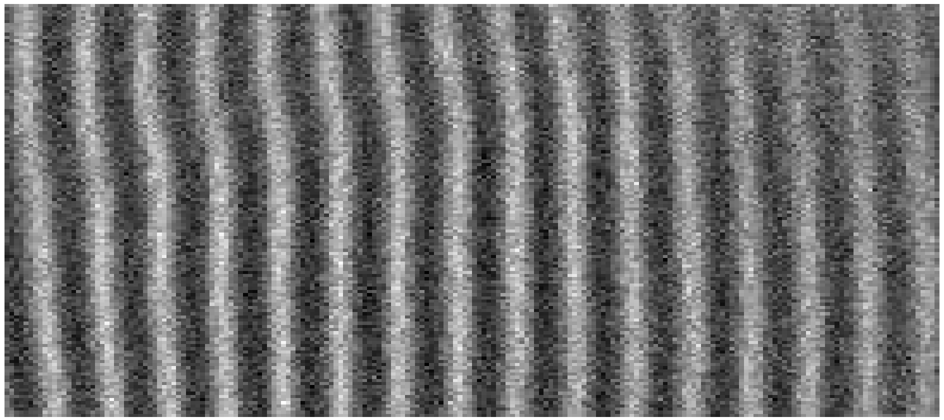
side view

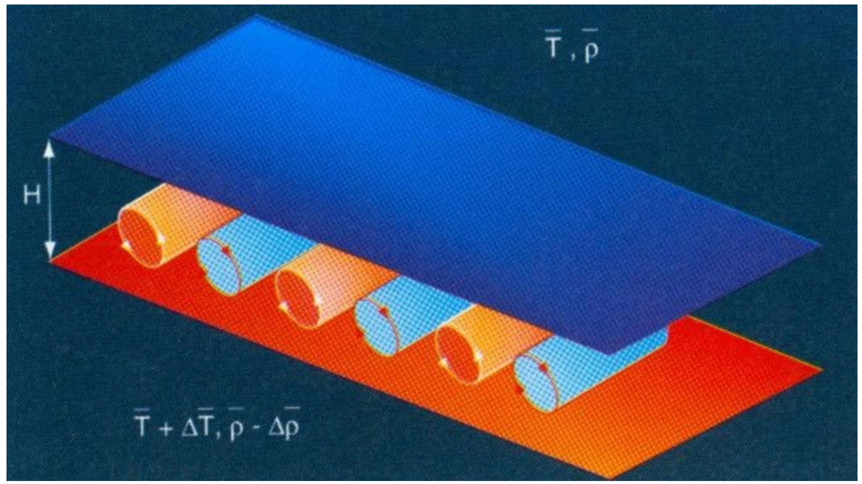


$T = T_1 < T_0$

$T = T_0$

top view





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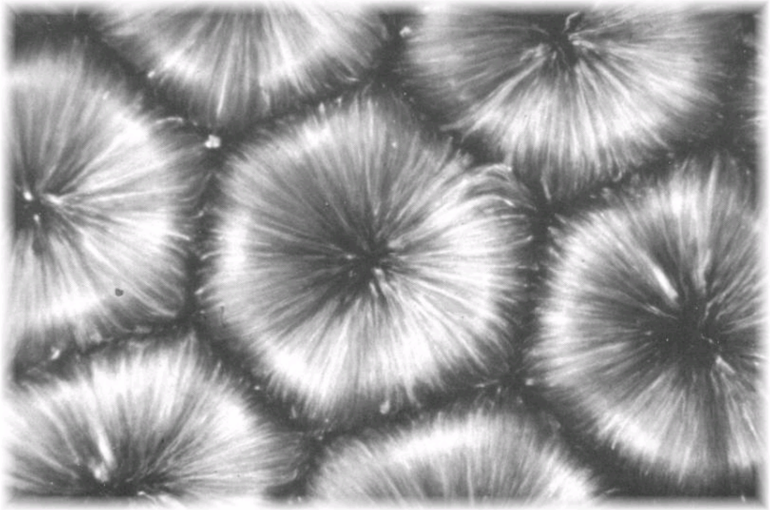
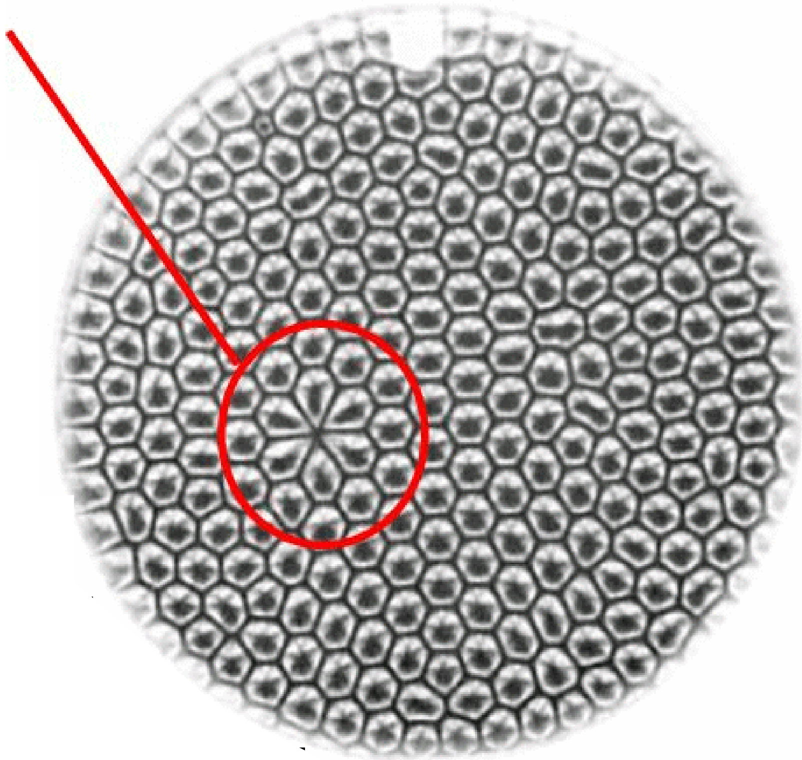
http://www.esam.northwestern.edu/~riecke/Vorlesungen/412/1999/rb.html

http://hmf.enseeiht.fr/travaux/CD0001/travaux/optmfn/hi/01pa/hyb72/rb/rb.htm

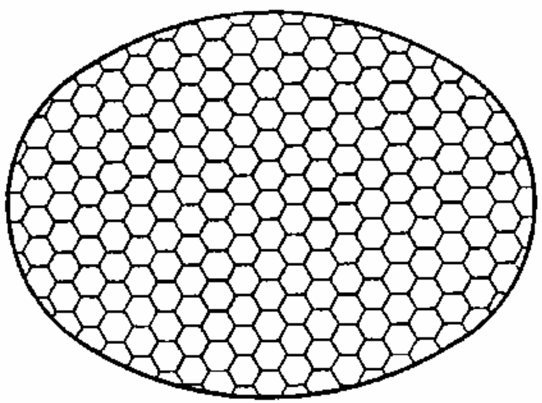
Rayleigh Bénard Convection



3D convection: hexagons
(with some defects)



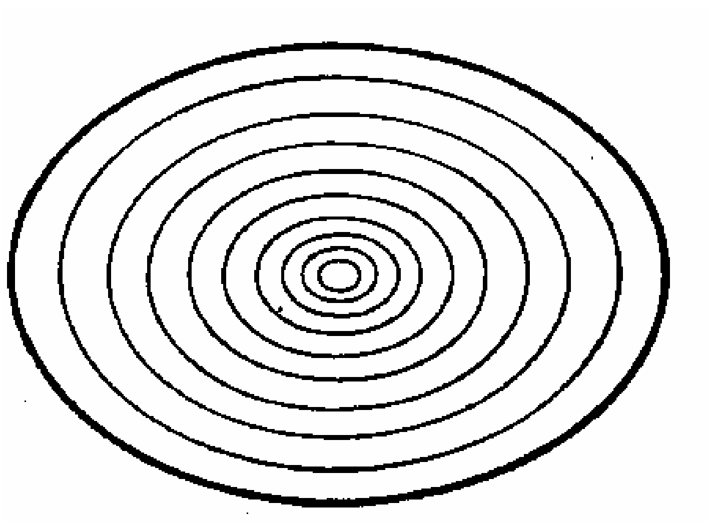
top views

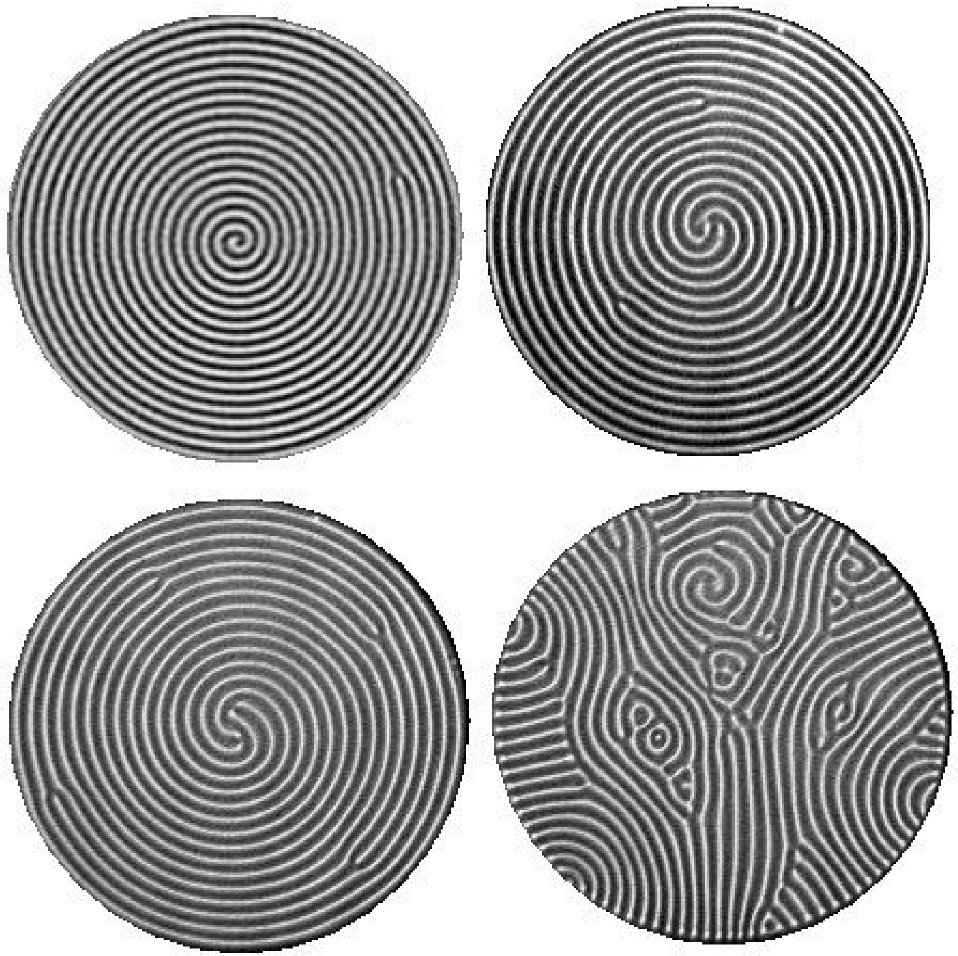


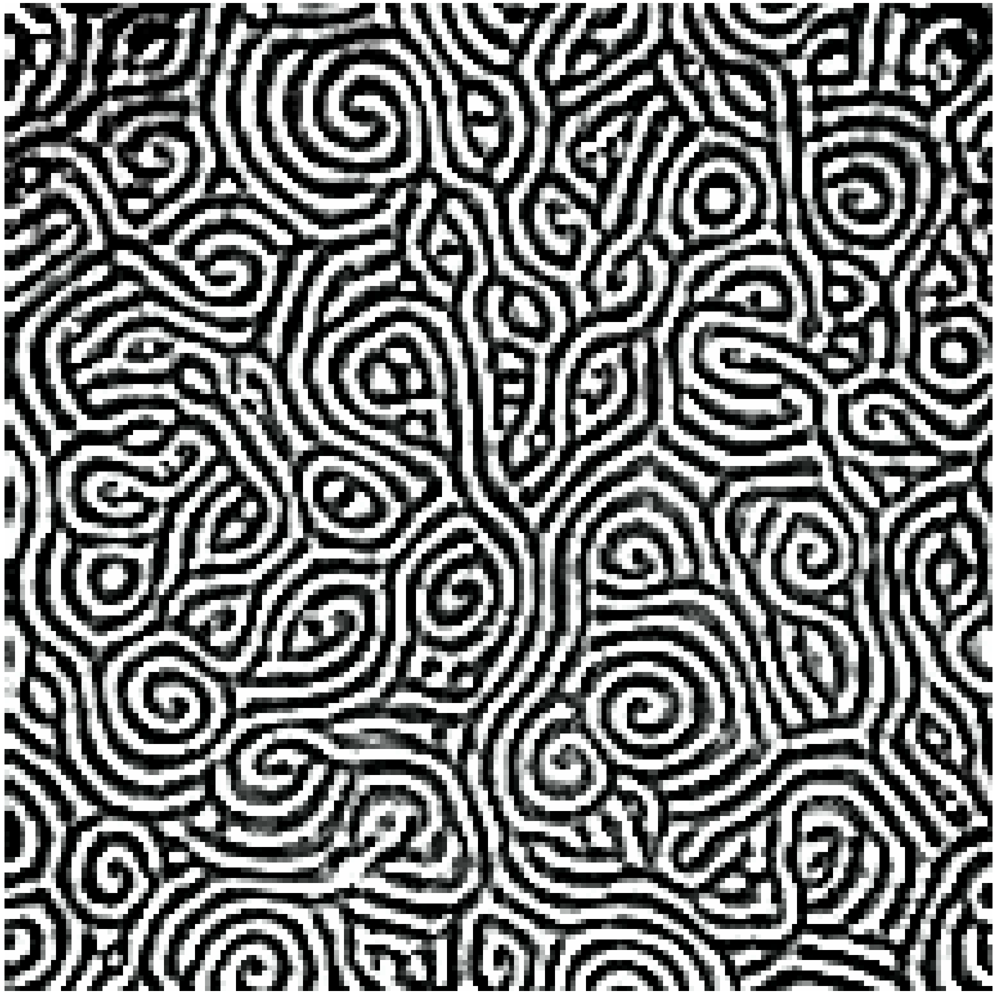
Rayleigh Bénard Convection

3D convections: more complicated patters can appear
S
pirals and some related patterns

top views







Defect chaos in Rayleigh-Benard convection

Stephen Morris at the University of Toronto http://cnls.lanl.gov/~nbt/movies.html

Rayleigh Bénard convection
Equilibrium Solution

$$\nabla \bullet \mathbf{v} = 0$$

$$\rho_0 \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{v} \right) = -\nabla p - \rho g \hat{\mathbf{z}} + \nu \Delta \mathbf{v}$$

$$\rho(T) = \rho_0 \left[1 - \alpha \left(T - T_0 \right) \right]$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \bullet \nabla T = \kappa \Delta T$$

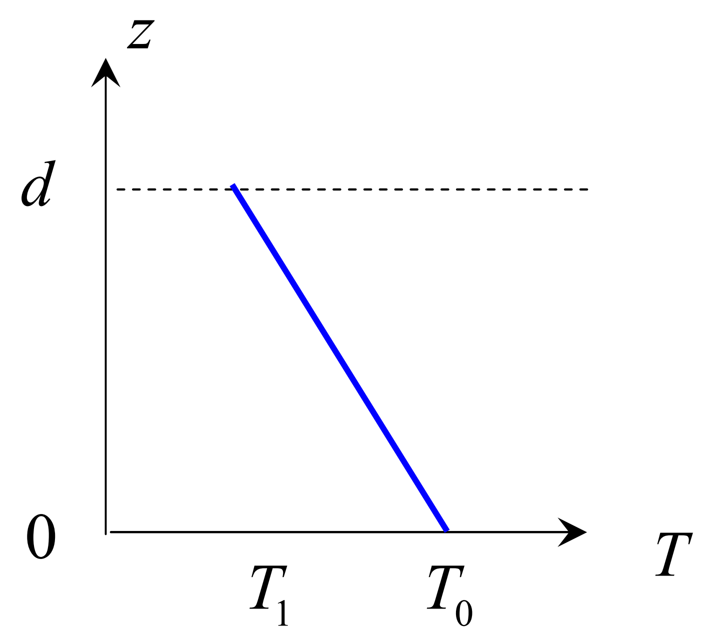
Set $\quad \mathbf{v} = 0, \quad \frac{\partial}{\partial t} = 0, \quad z$ dependent only

$$\left(\text{with} \quad \nu = \kappa = 0 \right)$$

Rayleigh Bénard convection
Equilibrium Solution

$$\overline{T} = T_0 - \beta z, \qquad \beta = \frac{T_0 - T_1}{d} \quad \text{(temperature gradient)}$$

$$\overline{p} = p_0 + \rho_0 g z \left(1 + \frac{\alpha\beta}{2} z \right)$$



Rayleigh Bénard convection equations for perturbation

By setting $\theta = T - \bar{T}(z) \quad \tilde{p} = p - \bar{p}(z)$

$$\nabla \bullet \mathbf{v} = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{v} = -\frac{\nabla p}{\rho_0} + g\alpha\theta\mathbf{z} + \nu \Delta \mathbf{v}$$

we have rewritten $\frac{\nu}{\rho_0} \rightarrow \nu$

$$\frac{\partial \theta}{\partial t} + \mathbf{v} \bullet \nabla \theta = \beta w + \kappa \Delta \theta$$

$w = \mathbf{v} \bullet \mathbf{z}$ (i.e., z component of $\mathbf{v}$)

$\theta(t, x, y, 0) = \theta(t, x, y, d) = 0$ (Boundary conditions)

Rayleigh Bénard convection normalized equations

By setting

$$\mathrm{v} \rightarrow \frac{\nu}{d}\mathrm{v}$$

$$\theta \rightarrow \frac{\beta d \nu}{\kappa}\theta$$

$$\tilde{p} \rightarrow \frac{\rho_0 \nu^2}{d^2} p$$

$$t \rightarrow \frac{d^2}{\kappa} t$$

$$\left(x, y, z\right) \rightarrow d\left(x, y, z\right)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{v} = -\nabla p + R\theta\hat{\mathbf{z}} + \Delta \mathbf{v}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{v} \bullet \nabla \theta = \frac{1}{P} w + \frac{1}{P}\Delta\theta$$

$$\nabla \bullet \mathbf{v} = 0$$

$R:$ Rayleigh number

$P:$ Prandtl number

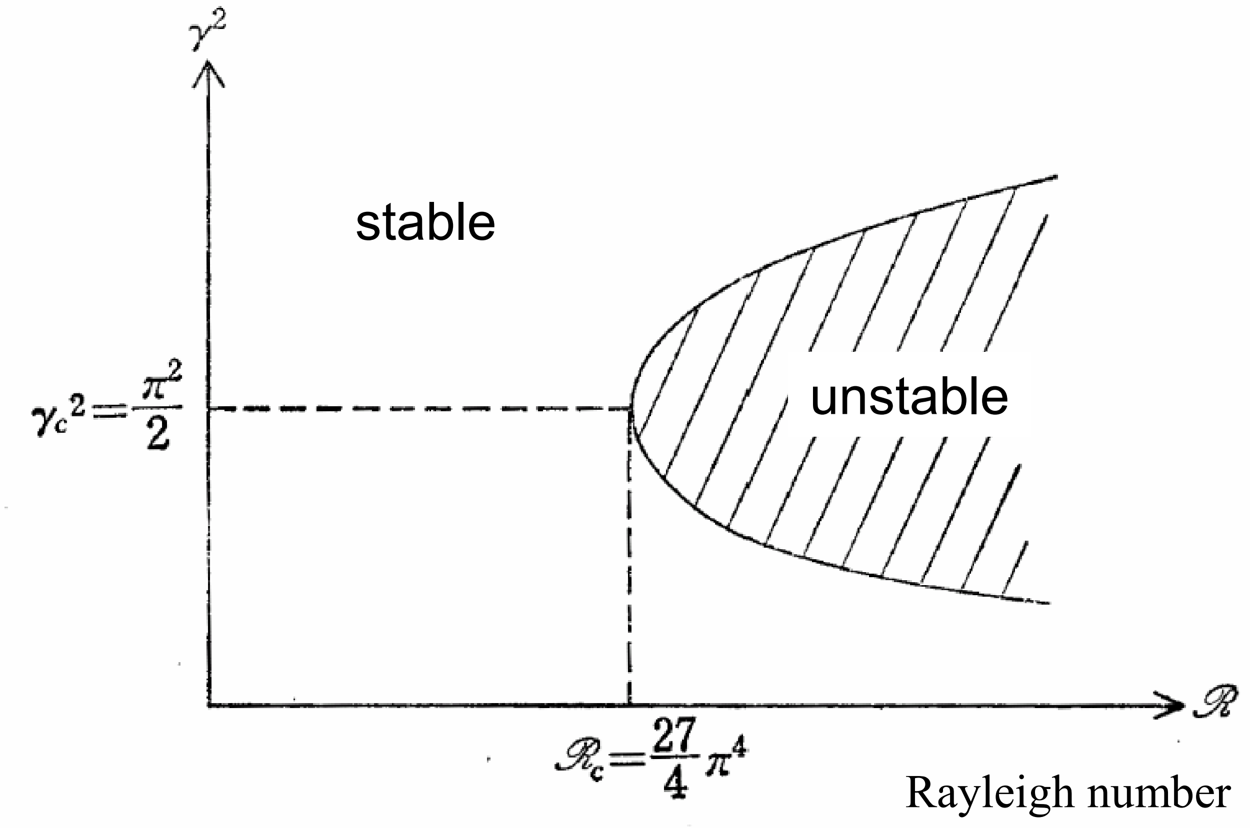
Rayleigh Bénard convection 2D normalized equations

Consider only 2D solutions:

$$\mathbf{v} = \nabla\phi(y,z)\times\hat{\mathbf{x}} = \left(0, \frac{\partial\phi}{\partial z}, -\frac{\partial\phi}{\partial y}\right)$$

$$\begin{cases} \dfrac{d}{dt}\Delta\phi = -R\dfrac{\partial\theta}{\partial y} + \Delta^2\phi \\[2ex] \dfrac{d\theta}{dt} = -\dfrac{1}{P}\dfrac{\partial\phi}{\partial y} + \dfrac{1}{P}\Delta\theta \end{cases}$$

Rayleigh Bénard convection
Stability Analysis



Rayleigh Bénard convection

Heat Transfer

Nusselt Number

$$Nu = \frac{\text{total heat transfer}}{\text{heat transfer at rest}} = \frac{H}{\kappa \rho_0 c_p (T_0 - T_1)/d}$$

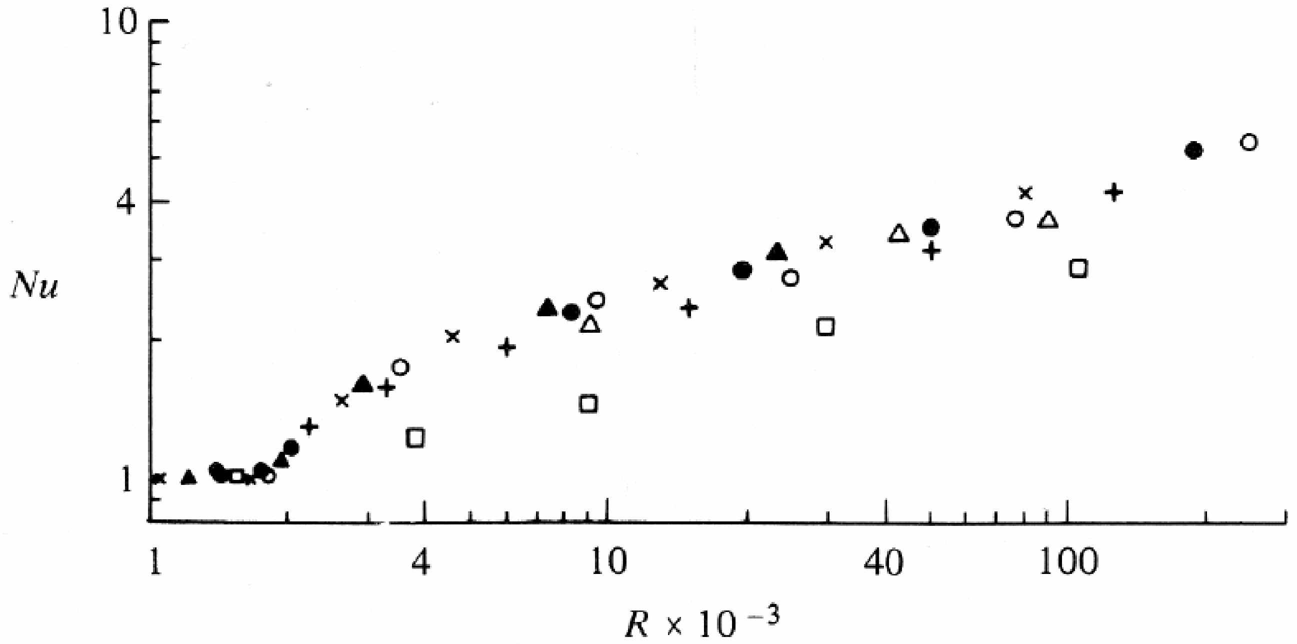


Fig. 2.6. Some experimental results on the heat transfer in various fluids in various containers. The Nusselt number is plotted against the Rayleigh number; ○ water; + heptane; × ethylene glycol; ● silicone oil AK 3; ▲ silicone oil AK 350; △ air; □ mercury. After Silveston 1958 and Rossby 1969.)

From "Hydrodynamic stability" (Drazin &Reid)

Resistive interchange mode

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\bullet\nabla\mathbf{v}\right)=-\nabla p+\mathbf{j}\times\mathbf{B}+\nabla\bullet\mathbf{\Pi}$$

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times\left(\mathbf{v}\times\mathbf{B}-\eta\mathbf{j}\right) \qquad \mu_0\mathbf{j}=\nabla\times\mathbf{B} \qquad \nabla\bullet\mathbf{B}=0$$

$$\frac{\partial \rho}{\partial t}+\nabla\bullet\left(\rho\mathbf{v}\right)=0$$

$$\frac{\partial p}{\partial t}+\mathbf{v}\bullet\nabla p+\gamma p\nabla\bullet\mathbf{v}=\left(\gamma-1\left[\eta\mathbf{j}\bullet\mathbf{j}+\nabla\bullet\left(\kappa\nabla T\right)-\mathbf{\Pi}:\nabla\mathbf{v}\right]\right)$$

$$p=\rho T$$

$$\mathbf{\Pi}=-3\mu_{\|}\left(\hat{\mathbf{b}}\hat{\mathbf{b}}-\frac{1}{3}\mathbf{I}\right)\lambda-\mu_{\perp}\sigma \qquad \lambda=\hat{\mathbf{b}}\bullet\left[\left(\hat{\mathbf{b}}\bullet\nabla\right)\mathbf{v}\right]-\frac{1}{3}\nabla\bullet\mathbf{v}$$

$$\sigma_{ij}=\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\nabla\bullet\mathbf{v}$$

Resistive interchange mode

Equations for the mean fields

$$\nabla p_0 = \mathbf{j}_0 \times \mathbf{B}_0$$

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times \left(\mathbf{u}_0 \times \mathbf{B}_0 + \boldsymbol{\varepsilon} - \eta \mathbf{j}_0 \right) \qquad \mu_0 \mathbf{j}_0 = \nabla \times \mathbf{B}_0 \quad \nabla \bullet \mathbf{B}_0 = 0$$

$$\frac{\partial \rho_0}{\partial t} + \nabla \bullet \left(\rho_0 \mathbf{u}_0 \right) = 0$$

$$\frac{\partial p_0}{\partial t} + \mathbf{u}_0 \bullet \nabla p_0 + \gamma p_0 \nabla \bullet \mathbf{u}_0 = \left(\gamma - 1 \left[\eta \mathbf{j}_0 \bullet \mathbf{j}_0 + \nabla \bullet \left(\kappa \nabla T_0 \right) - \nabla \bullet \left(p_0 \langle s_1 \mathbf{v}_1 \rangle \right) + f \right] \right)$$

$$\mathbf{u}_0 = \mathbf{v}_0 + \left(1/\rho_0 \right) \langle \rho_1 \mathbf{v}_1 \rangle \qquad s \equiv \left[1/\left(\gamma - 1 \right) \right] \log \left(p/\rho^\gamma \right)$$

$$\boldsymbol{\varepsilon} = \langle \mathbf{v}_1 \times \mathbf{B}_1 \rangle - \left(1/\rho_0 \right) \langle \rho_1 \mathbf{v}_1 \rangle \times \mathbf{B}_0$$

$$f = -\nabla \bullet \left\langle \left(p_1 + \mathbf{B}_0 \bullet \mathbf{B}_1 \right) \mathbf{v}_1 - \left(\mathbf{v}_1 \bullet \mathbf{B}_1 \right) \mathbf{B}_0 + \left(1/2 \right) \rho_0 \mathbf{v}^2 \bullet \mathbf{v} \right\rangle$$

Resistive interchange mode

Equations for fluctuating fields

$$\mathbf{B}_1 = \nabla_\perp A \times \mathbf{b} - p_1 \mathbf{b}$$

$$\mathbf{j}_1 = -\left(\nabla_\perp^2 A\right)\mathbf{b} - \nabla_\perp p_1 \times \mathbf{b}$$

$$\mathrm{v}_1 = \nabla_\perp \phi \times \mathbf{b} - v\mathbf{b}$$

$$\nabla_\perp \equiv \nabla - \mathbf{b}\left(\mathbf{B}_0 \nabla \cdot\right) \qquad \mathbf{b} = \mathbf{B}_0 \big/ \left|\mathbf{B}_0\right|^2$$

Resistive interchange mode

Equations for fluctuating fields

$$\frac{d\tilde{A}}{d\tau} = \frac{\partial\tilde{\phi}}{\partial\tilde{\theta}} + \frac{1}{R}\Delta_\perp \tilde{A}$$

$$\frac{d}{d\tau}\Delta_\perp\tilde{\phi} = \frac{\partial}{\partial\tilde{\theta}}\Delta_\perp\tilde{A} + \left\{\tilde{A}, \Delta_\perp\tilde{A}\right\} - \frac{\partial\tilde{p}}{\partial y} + M\Delta_\perp^2\tilde{\phi}$$

$$\frac{d\tilde{p}}{d\tau} = -D\frac{\partial\tilde{\phi}}{\partial y} + \chi\Delta_\perp\tilde{p}$$

$$\frac{d\tilde{v}}{d\tau} = \frac{\partial\tilde{p}}{\partial\tilde{\theta}} + \left\{\tilde{A}, \tilde{p}\right\} + D\frac{\partial\tilde{A}}{\partial y} + M\Delta_\perp\tilde{v}$$

$$\frac{d}{d\tau} = \frac{\partial}{\partial\tau} + \left\{\phi, \quad\right\}$$

$$D \propto -p_0' = -\left.\frac{dp_0}{dr}\right|_{r=r_0}$$

Resistive interchange mode

Normalization

$$x = |\sigma|^{1/2}\left[(r - r_0)/r_0\right]$$

$$y = (B_\theta / r_0 B)|\sigma|^{1/2}\left[z - \mu(r)\theta\right]$$

$$\tilde{\theta} = |\sigma|\theta$$

$$\tau = \left(B_\theta |\sigma| / r_0 \sqrt{\rho_0}\right)t$$

$$\mu(r) = \frac{rB_z}{B_\theta} \quad \sigma = \frac{B_\theta \mu'}{B}$$

parameters are all evaluated at $r = r_0$

Resistive interchange mode

$$\frac{d\tilde{A}}{d\tau} = \frac{\partial\tilde{\phi}}{\partial\tilde{\theta}} + \frac{1}{R}\Delta_\perp\tilde{A}$$

$$\frac{d}{d\tau}\Delta_\perp\tilde{\phi} = \frac{\partial}{\partial\tilde{\theta}}\Delta_\perp\tilde{A} + \left\{\tilde{A}, \Delta_\perp\tilde{A}\right\} - \frac{\partial\tilde{p}}{\partial y} + M\Delta_\perp^2\tilde{\phi}$$

$$\frac{d\tilde{p}}{d\tau} = -D\frac{\partial\tilde{\phi}}{\partial y} + \chi\Delta_\perp\tilde{p}$$

$$\frac{d}{d\tau} = \frac{\partial}{\partial\tau} + \left\{\phi, \quad\right\} \qquad D \propto -p_0' = -\left.\frac{dp_0}{dr}\right|_{r=r_0}$$

Resistive interchange mode

electrostatic limit

$$\frac{d\tilde{A}}{d\tau} = \frac{\partial \tilde{\phi}}{\partial \tilde{\theta}} + \frac{1}{R}\Delta_\perp \tilde{A}$$

$$\frac{d}{d\tau}\Delta_\perp \tilde{\phi} = \frac{\partial}{\partial \tilde{\theta}}\Delta_\perp \tilde{A} + \left\{\tilde{A}, \Delta_\perp \tilde{A}\right\} - \frac{\partial \tilde{p}}{\partial y} + M\Delta_\perp^2 \tilde{\phi}$$

$$\frac{d\tilde{p}}{d\tau} = -D\frac{\partial \tilde{\phi}}{\partial y} + \chi \Delta_\perp \tilde{p}$$

$$D \propto -p_0' = -\left.\frac{dp_0}{dr}\right|_{r=r_0}$$

Resistive interchange mode

electrostatic limit

$$\frac{d}{d\tau}\Delta_\perp\tilde{\phi} = -R\frac{\partial^2}{\partial\tilde{\theta}^2}\tilde{\phi} - \frac{\partial\tilde{p}}{\partial y} + M\Delta_\perp^2\tilde{\phi}$$

$$\frac{d\,\tilde{p}}{d\tau} = -D\frac{\partial\tilde{\phi}}{\partial y} + \chi\Delta_\perp\tilde{p}$$

Resistive interchange mode

electrostatic limit

$$\frac{d}{d\tau}\Delta_\perp\tilde{\phi} = -R\frac{\partial^2}{\partial\tilde{\theta}^2}\tilde{\phi} - \frac{\partial\tilde{p}}{\partial y} + M\Delta_\perp^2\tilde{\phi}$$

$$\frac{d\,\tilde{p}}{d\tau} = -D\frac{\partial\tilde{\phi}}{\partial y} + \chi\Delta_\perp\tilde{p}$$

2D Rayleigh Bénard Convection

$$\begin{cases} \dfrac{d}{dt}\Delta\phi = -R\dfrac{\partial\theta}{\partial y} + \Delta^2\phi \\[2ex] \dfrac{d\theta}{dt} = -\dfrac{1}{P}\dfrac{\partial\phi}{\partial y} + \dfrac{1}{P}\Delta\theta \end{cases}$$

Resistive interchange mode

Alfvén wave

$$\frac{d\tilde{A}}{d\tau} = \frac{\partial\tilde{\phi}}{\partial\tilde{\theta}} + \frac{1}{R}\Delta_\perp\tilde{A}$$

$$\frac{d}{d\tau}\Delta_\perp\tilde{\phi} = \frac{\partial}{\partial\tilde{\theta}}\Delta_\perp\tilde{A} + \left\{\tilde{A},\Delta_\perp\tilde{A}\right\} - \frac{\partial\tilde{p}}{\partial y} + M\Delta_\perp^2\tilde{\phi}$$

$$\frac{d\tilde{p}}{d\tau} = -D\frac{\partial\tilde{\phi}}{\partial y} + \chi\Delta_\perp\tilde{p}$$

Resistive interchange mode

Tearing mode

$$\frac{d\tilde{A}}{d\tau} = \frac{\partial \tilde{\phi}}{\partial \tilde{\theta}} + \frac{1}{R}\Delta_{\perp}\tilde{A}$$

$$\frac{d}{d\tau}\Delta_{\perp}\tilde{\phi} = \frac{\partial}{\partial \tilde{\theta}}\Delta_{\perp}\tilde{A} + \left\{\tilde{A}, \Delta_{\perp}\tilde{A}\right\} - \frac{\partial \tilde{p}}{\partial y} + M\Delta_{\perp}^{2}\tilde{\phi}$$

$$\frac{d\tilde{p}}{d\tau} = -D\frac{\partial \tilde{\phi}}{\partial y} + \chi\Delta_{\perp}\tilde{p}$$

Resistive interchange mode

bifurcation analysis

$$\frac{d\tilde{A}}{d\tau} = \frac{\partial \tilde{\phi}}{\partial \tilde{\theta}} + \frac{1}{R}\Delta_\perp \tilde{A}$$

$$\frac{d}{d\tau}\Delta_\perp \tilde{\phi} = \frac{\partial}{\partial \tilde{\theta}}\Delta_\perp \tilde{A} + \left\{\tilde{A}, \Delta_\perp \tilde{A}\right\} - \frac{\partial \tilde{p}}{\partial y} + M\Delta_\perp^2 \tilde{\phi}$$

$$\frac{d\tilde{p}}{d\tau} = -D\frac{\partial \tilde{\phi}}{\partial y} + \chi \Delta_\perp \tilde{p}$$

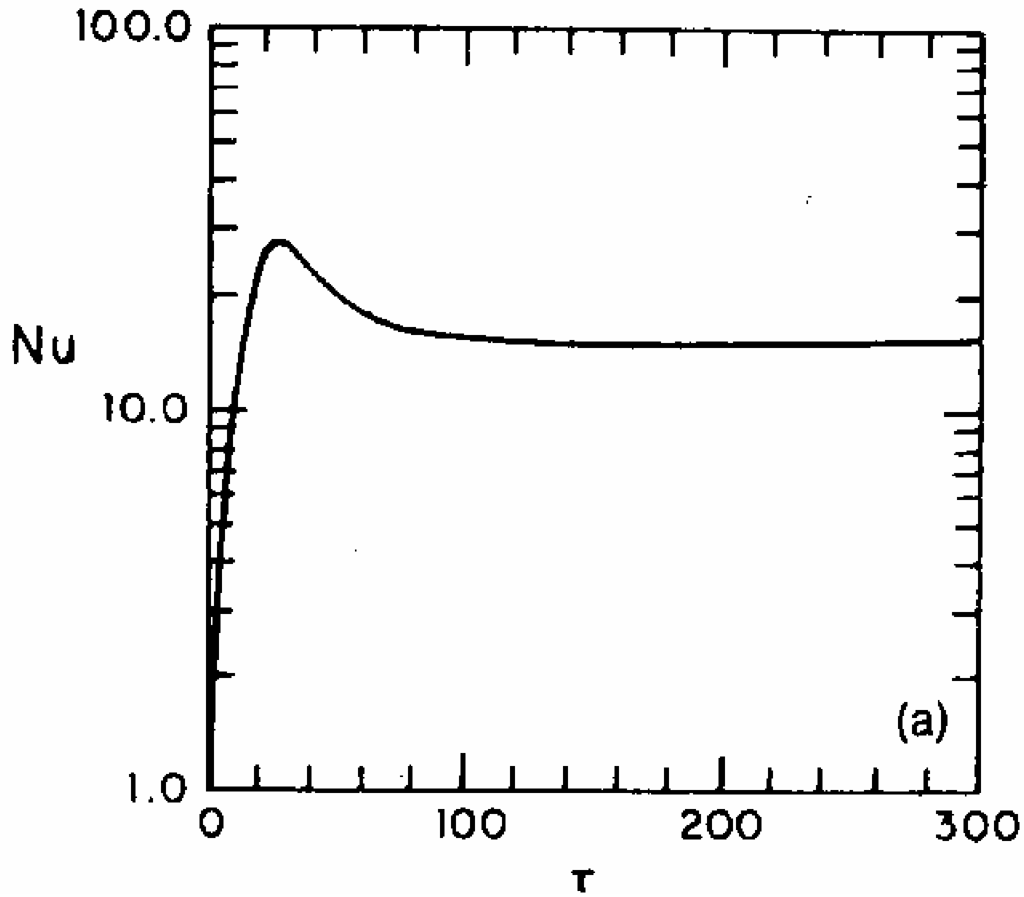
Consider the case $\frac{\partial}{\partial t} = 0$: steady state (i.e., mode saturation)

Resistive interchange mode

2D numerical simulation

$D = 0.2 \quad M = 1 \quad K = 5 \times 10^{-3}$

$D < 0.25$: ideal stability limit by Suydam



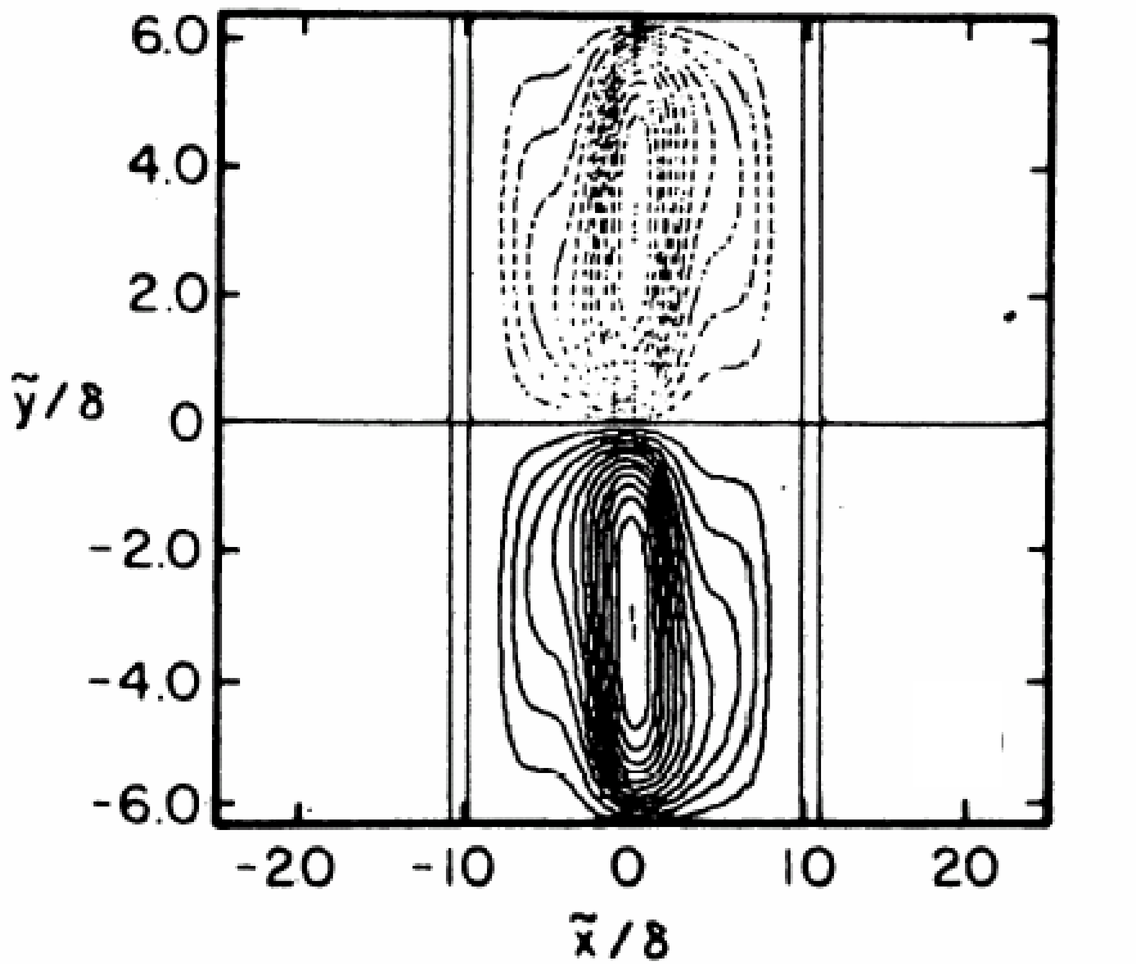
Nusselt Number $$Nu = 1 + p_0 \langle s_1 v_{1r} \rangle / -\kappa_\perp T_0'$$

Resistive interchange mode

2D numerical simulation

Contours of

$$\tilde{\phi}(\tilde{x},\tilde{y})$$



$D = 0.2 \quad M = 1 \quad K = 5\times10^{-3}$

bifurcation

From the obvious equilibrium $\tilde{A} = \tilde{\phi} = \tilde{p} = 0$ to another equilibrium

$$\frac{\partial}{\partial \tilde{\theta}} = \frac{\partial}{\partial z} + x\frac{\partial}{\partial y} \quad \rightarrow \quad x\frac{\partial}{\partial y} \qquad \text{in 2D}$$

$$\varepsilon^2 = \frac{1}{2}\left\langle \left|\nabla_\perp \tilde{\phi}\right|^2 \right\rangle$$

$$\tilde{A} = \varepsilon \hat{A} \quad \tilde{\phi} = \varepsilon \hat{\phi} \quad \tilde{p} = \varepsilon \hat{p}$$

Resistive interchange mode

bifurcation analysis

$$\varepsilon\left\{\hat{\phi},\hat{A}\right\}=\tilde{x}\frac{\partial\hat{\phi}}{\partial\tilde{y}}+\frac{1}{R}\Delta_{\perp}\hat{A}$$

$$\varepsilon\left\{\hat{\phi},\Delta_{\perp}\hat{\phi}\right\}=\tilde{x}\frac{\partial}{\partial\tilde{y}}\Delta_{\perp}\hat{A}+\left\{\hat{A},\Delta_{\perp}\hat{A}\right\}-\frac{\partial\hat{p}}{\partial y}+M\Delta_{\perp}^{2}\hat{\phi}$$

$$\varepsilon\left\{\hat{\phi},\hat{p}\right\}=-D\frac{\partial\hat{\phi}}{\partial y}+\chi\Delta_{\perp}\hat{p}$$

Resistive interchange mode

bifurcation analysis

$$\hat{A} = \sum_{n=0}^{\infty} \hat{A}_n \varepsilon^n, \quad \cdots$$

$$D = D_L + \sum_{n=0}^{\infty} D_n \varepsilon^n$$

$$L\begin{pmatrix} \hat{\phi}_0 \\ \hat{p}_0 \end{pmatrix} \equiv \begin{pmatrix} -R\tilde{x}^2\, \partial^2 \hat{\phi}_0 \big/ \partial \tilde{y}^2 - \partial \hat{p}_0 \big/ \partial y + M \Delta_\perp^2 \hat{\phi}_0 \\ \partial \hat{\phi}_0 \big/ \partial \tilde{y} - \left(\chi / D_L \right) \Delta_\perp \hat{p}_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Resistive interchange mode

bifurcation analysis

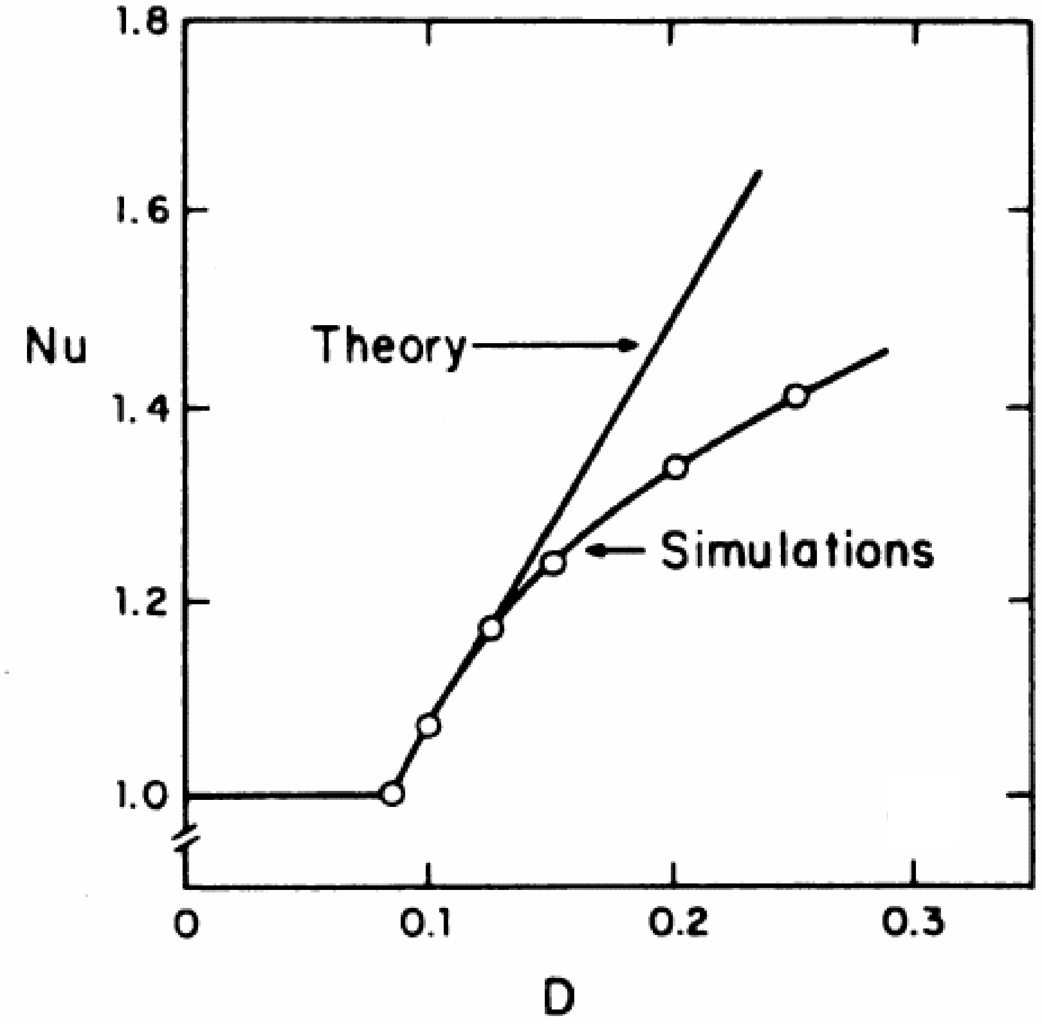
$$D_1 = 0 \qquad D = D_L + D_2 \varepsilon^2 + \cdots$$

$$p_0 \left\langle s_1 v_{1r} \right\rangle = -\frac{B_\theta B^2 \sigma^2}{(\gamma - 1)\sqrt{\rho_0}} k \left\langle \hat{p}_{01} \hat{\phi}_{01} \right\rangle \frac{D - D_L}{D_2}$$

Nusselt Number $$Nu = 1 + p_0 \left\langle s_1 v_{1r} \right\rangle \big/ -\kappa_\perp T_0'$$

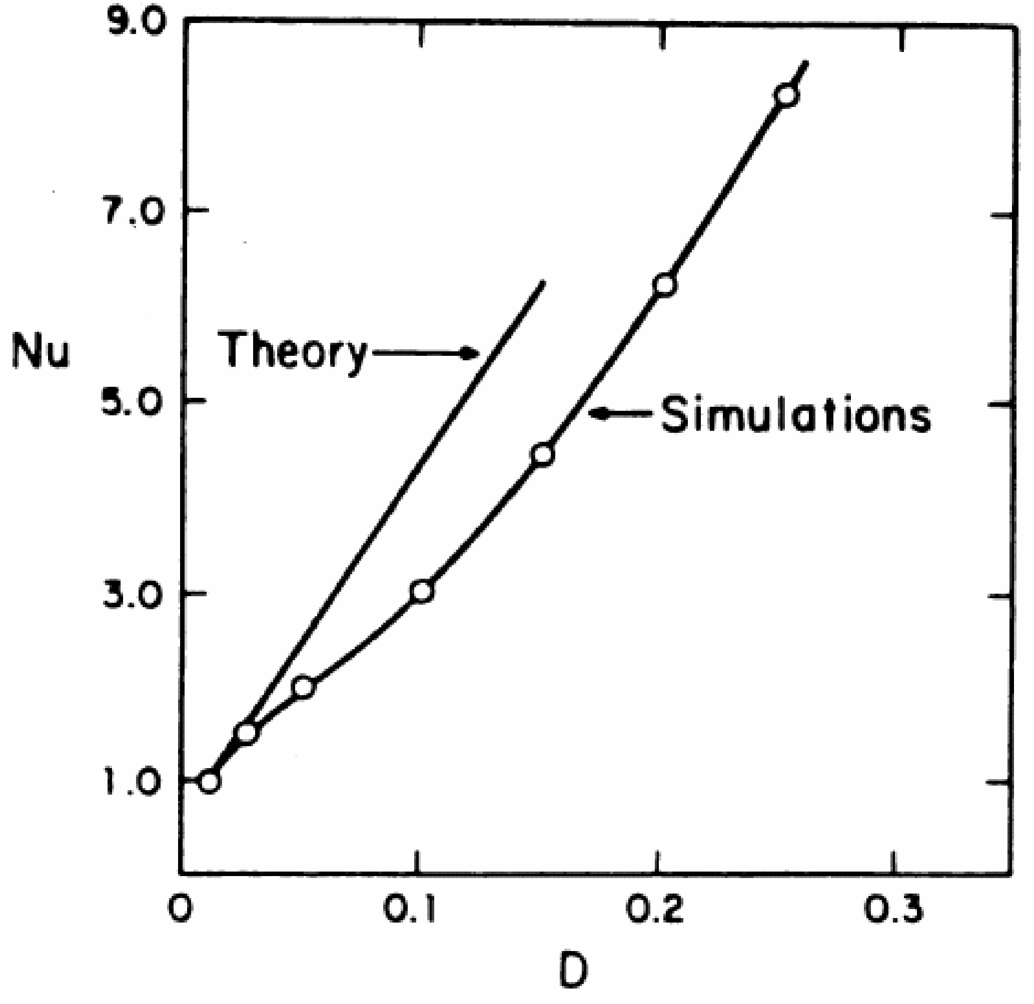
Resistive interchange mode

bifurcation analysis and 2D nonlinear simulation



Resistive interchange mode

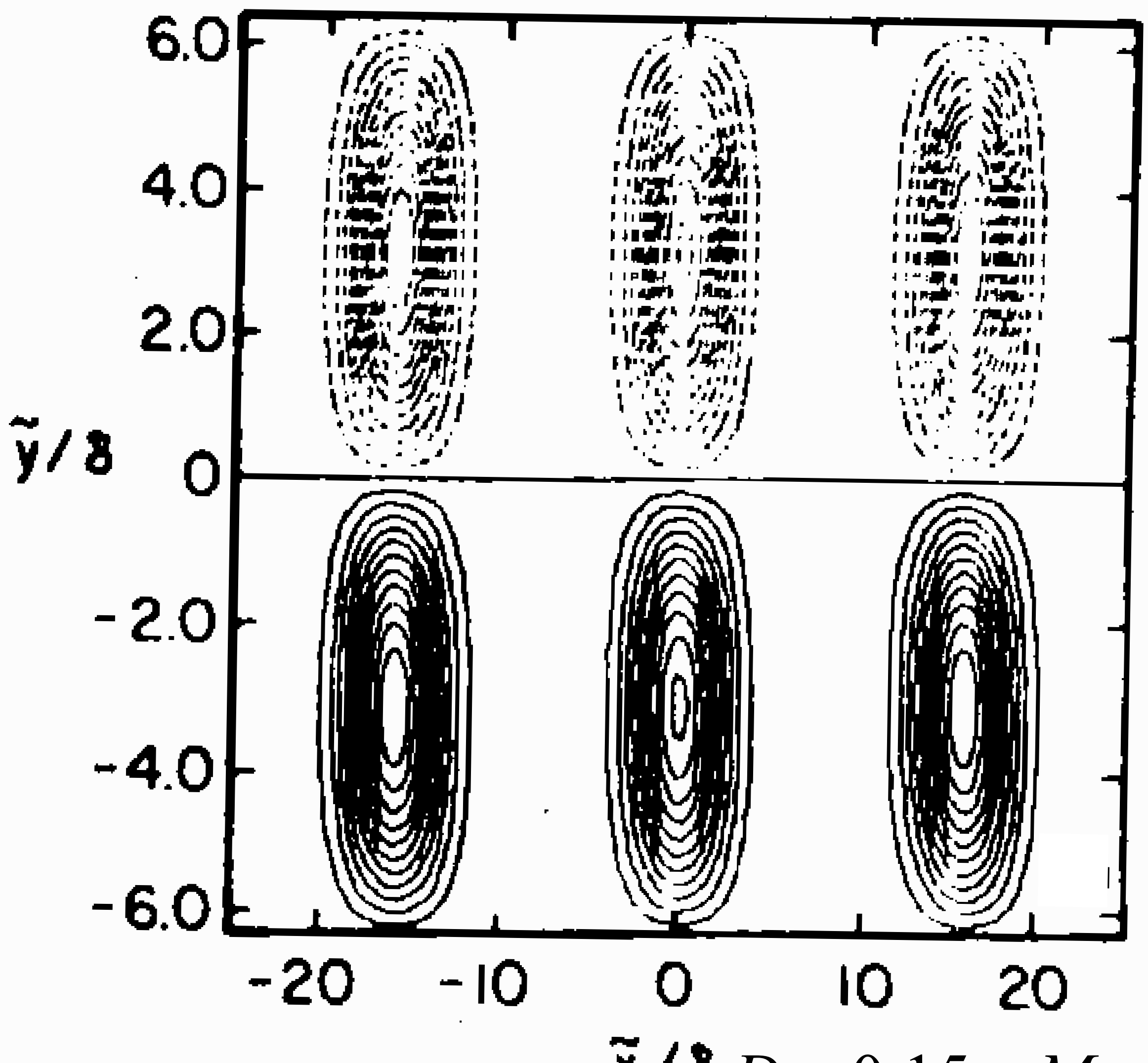
bifurcation analysis and 2D nonlinear simulation



$M = 1 \quad R = 10^{-2}$

Resistive interchange mode

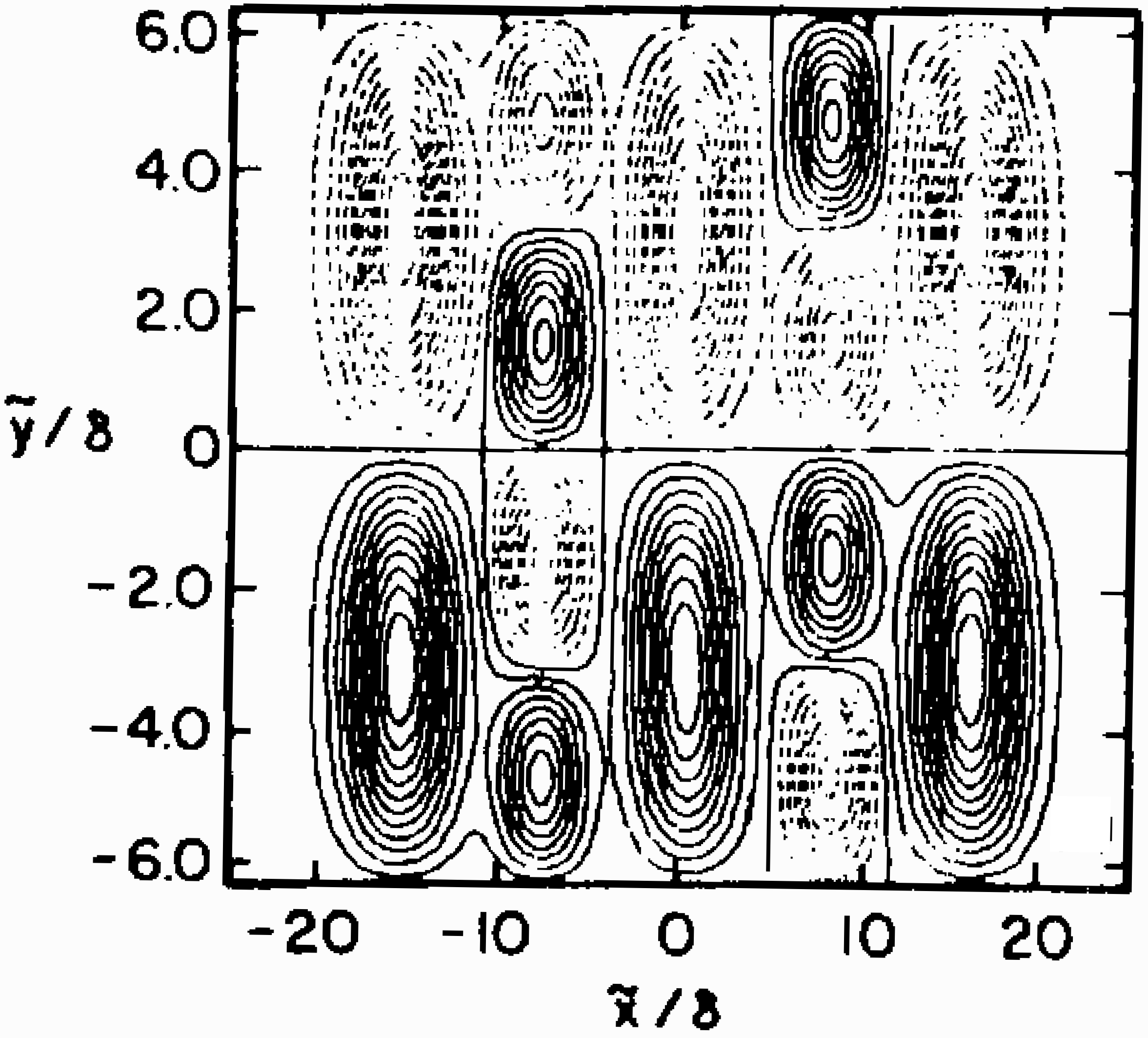
3D nonlinear simulation



$D = 0.15 \quad M = 1 \quad K = 6\times10^{-2}$

Resistive interchange mode

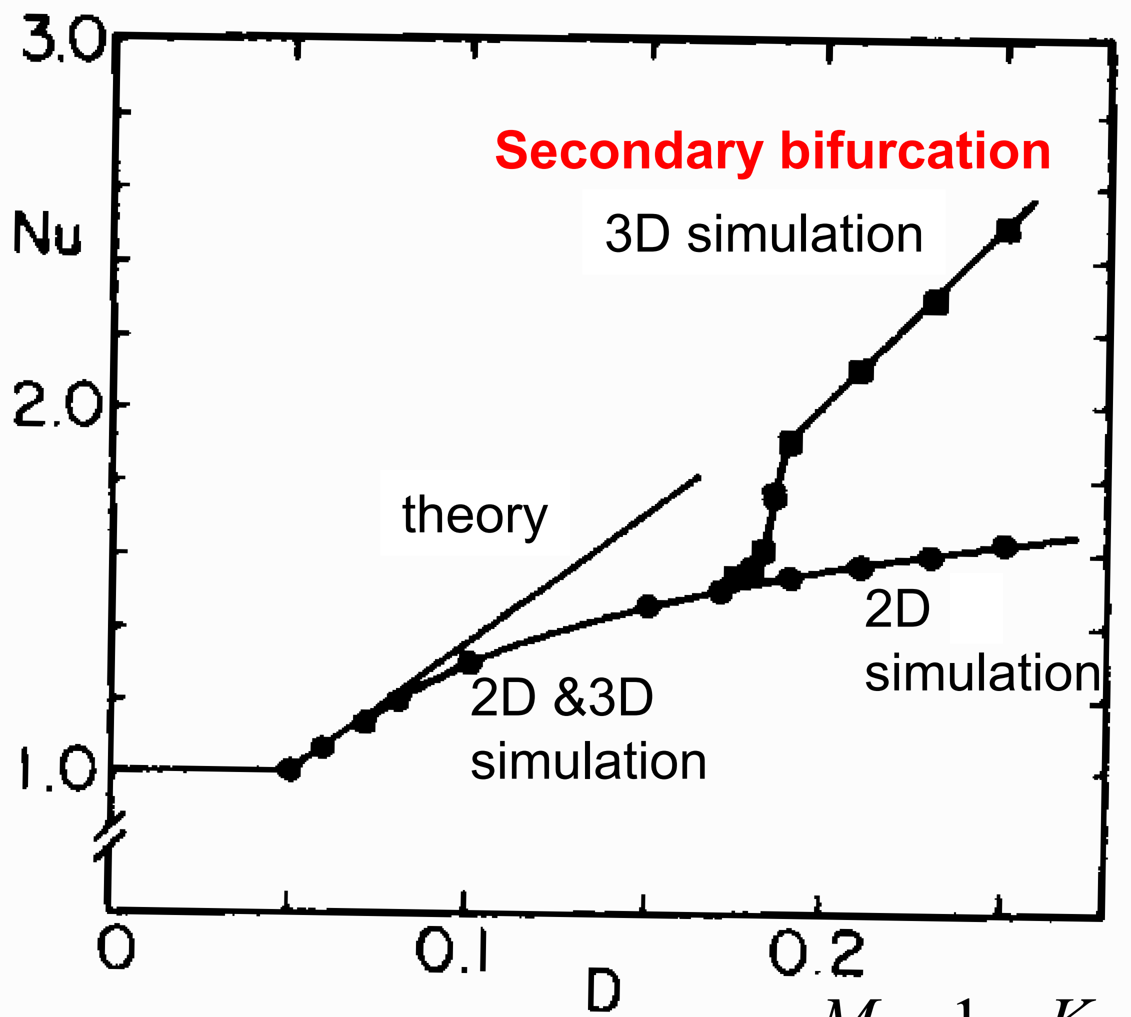
3D nonlinear simulation



$D = 0.23 \quad M = 1 \quad K = 6\times10^{-2}$

Resistive interchange mode

Comparison btw 2D and 3D nonlinear simulations



$M = 1 \quad K = 6 \times 10^{-2}$

practical issues for resistive interchange modes and resistive ballooning modes

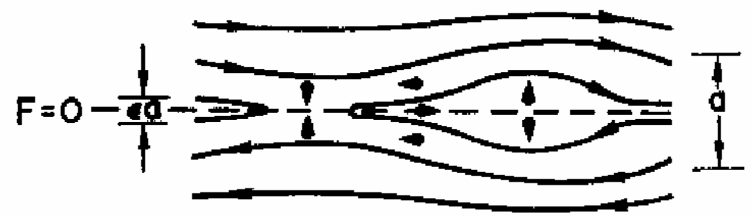
- strongly nonlinear evolution and turbulence
- effects of sheared flows on nonlinear modes
- flow generation from turbulence
- intermittent bursts of transport
- …

Nonlinear evolution of tearing modes with multiple rational surfaces

Tearing mode equations

$$\frac{d\tilde{A}}{d\tau} = \frac{\partial\tilde{\phi}}{\partial\tilde{\theta}} + \frac{1}{R}\Delta_{\perp}\tilde{A}$$

$$\frac{d}{d\tau}\Delta_{\perp}\tilde{\phi} = \frac{\partial}{\partial\tilde{\theta}}\Delta_{\perp}\tilde{A} + \left\{\tilde{A},\Delta_{\perp}\tilde{A}\right\} + M\Delta_{\perp}^{2}\tilde{\phi}$$



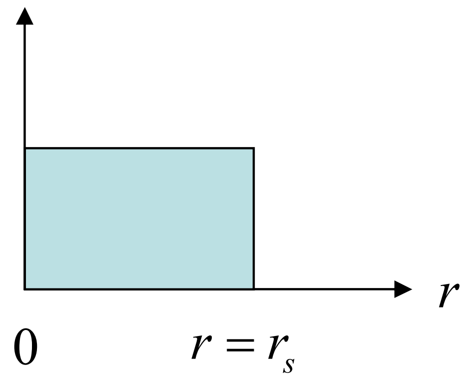
Furth, Killeen, & Rosenbluth, Phys. Fluids, 6, 459 (1963)

tearing mode

"m=1 tearing mode", "resistive kink mode"

$$\gamma = \left[q'(r_s) k r_s^2 S \right]^{2/3} \tau_R^{-1} \propto \eta^{1/3} q'^{2/3}$$

mode structure



$$q(r) = \frac{rB_t}{RB_p}$$ safety factor

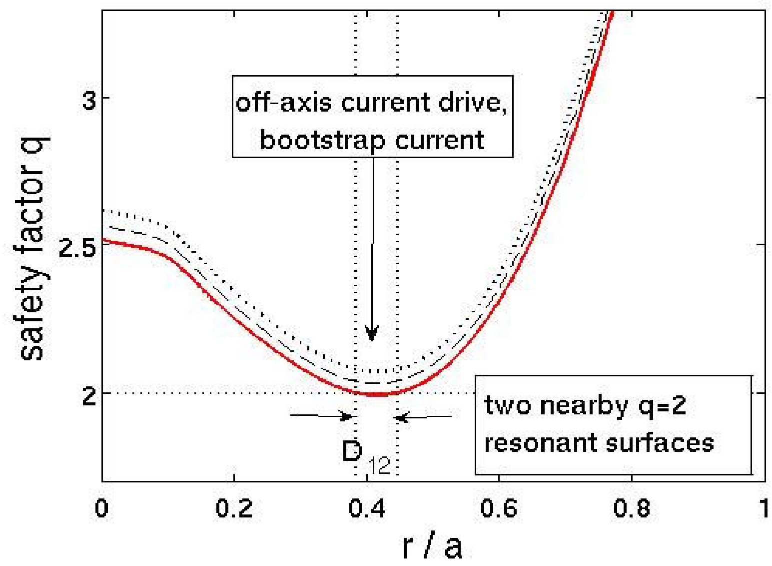
$$S = \tau_R / \tau_A$$ magnetic Reynolds number

$$\tau_R = a^2 / \eta$$ resistive skin time

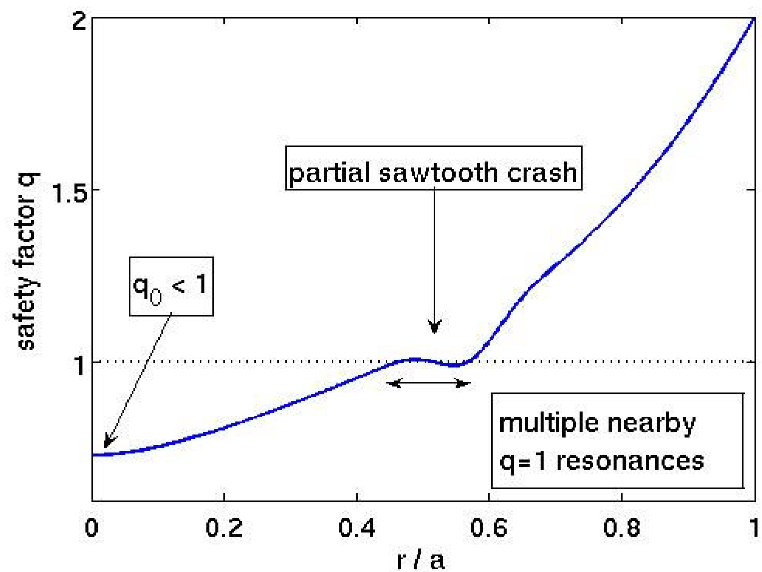
Alfvén transit time $$\tau_A = a/V_A \quad V_A = B_t / \sqrt{\rho_0}$$

multiple tearing modes

Multiple q=m/n resonant surfaces
may occur with a small distance apart



←Advanced reversed-shear tokamak & two resonant surfaces near $q_{\min}$

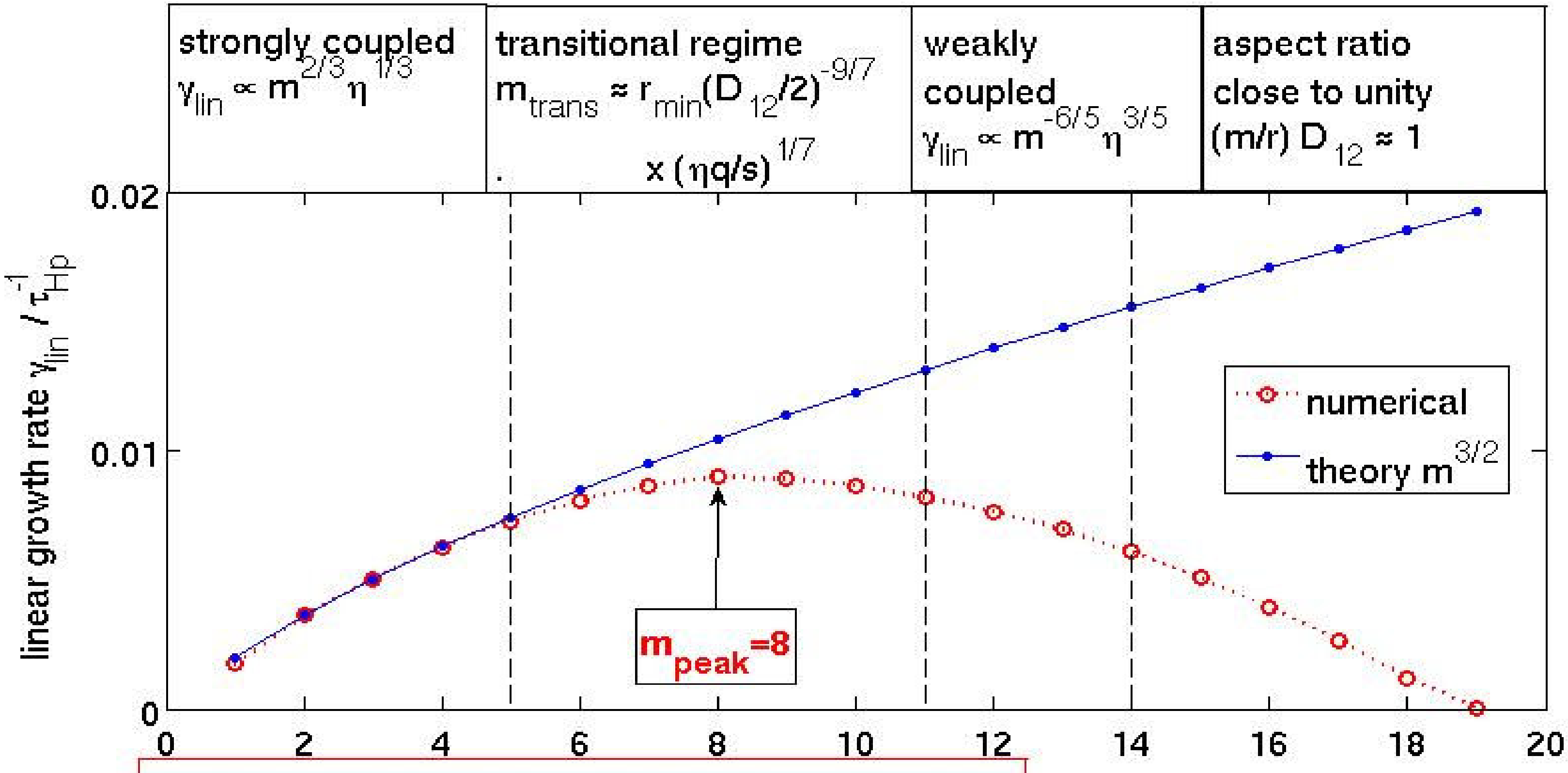


←Partial sawtooth collapse with q_0<1 and q~1 in an annular region

Multiple Tearing Modes (MTM)
may become unstable

Linear stability analyses:
growth rate vs. poloidal mode number

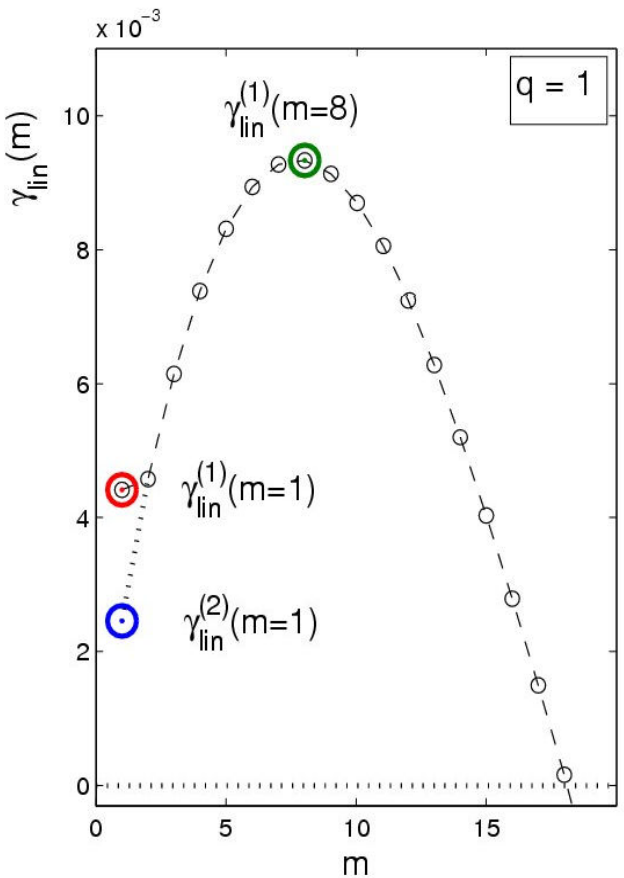
resistive q=1 Double Tearing Mode (DTM)

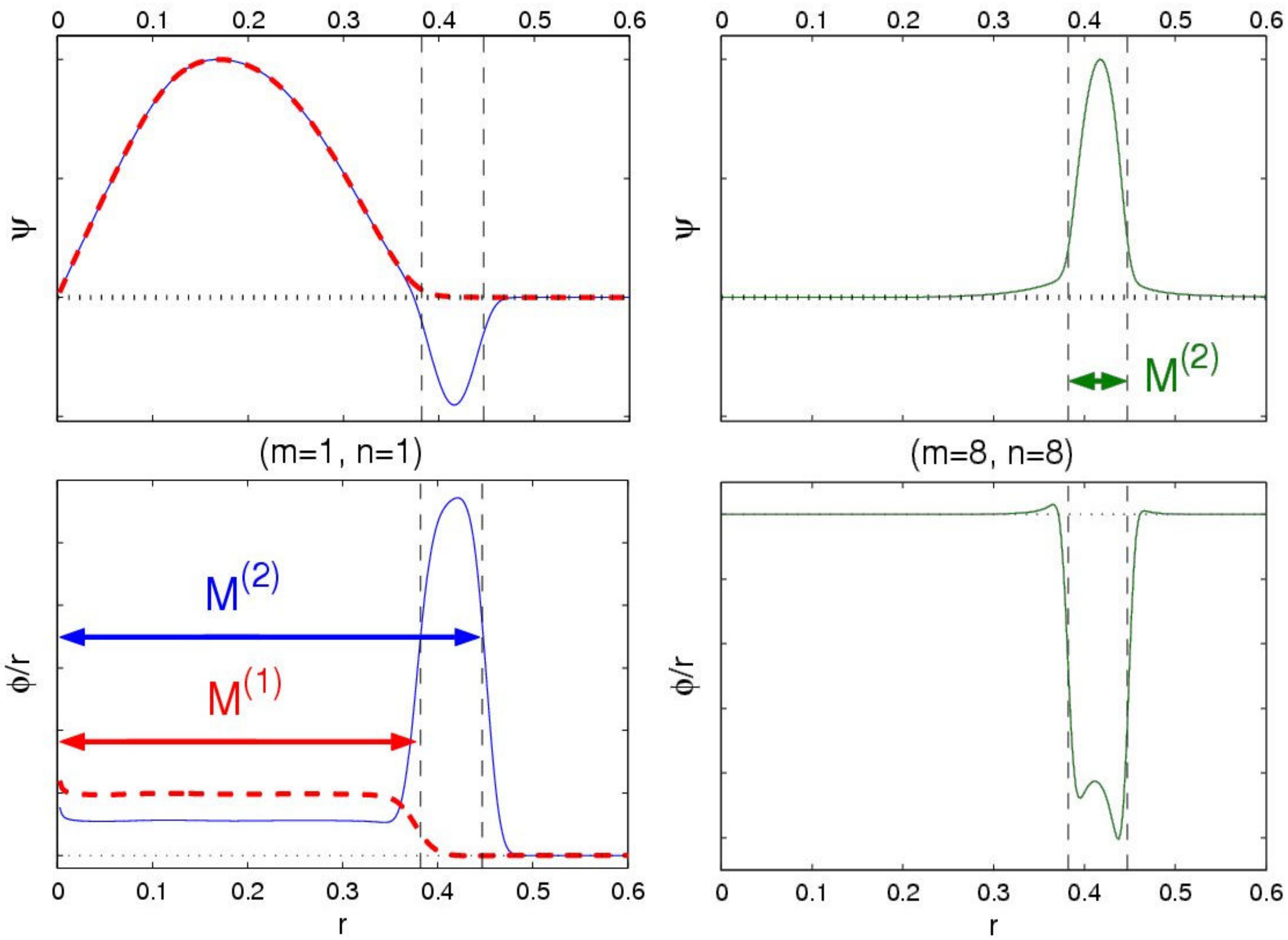


comparison between theory and numerical results yields semi-empiral estimate: $m_{peak} \approx m_{trans} + 1$

- *Theory [Pritchett et al. 1980] and numerical results agree*
- *Broad spectrum, dominant high-m modes*

Linear stability analyses: mode structures

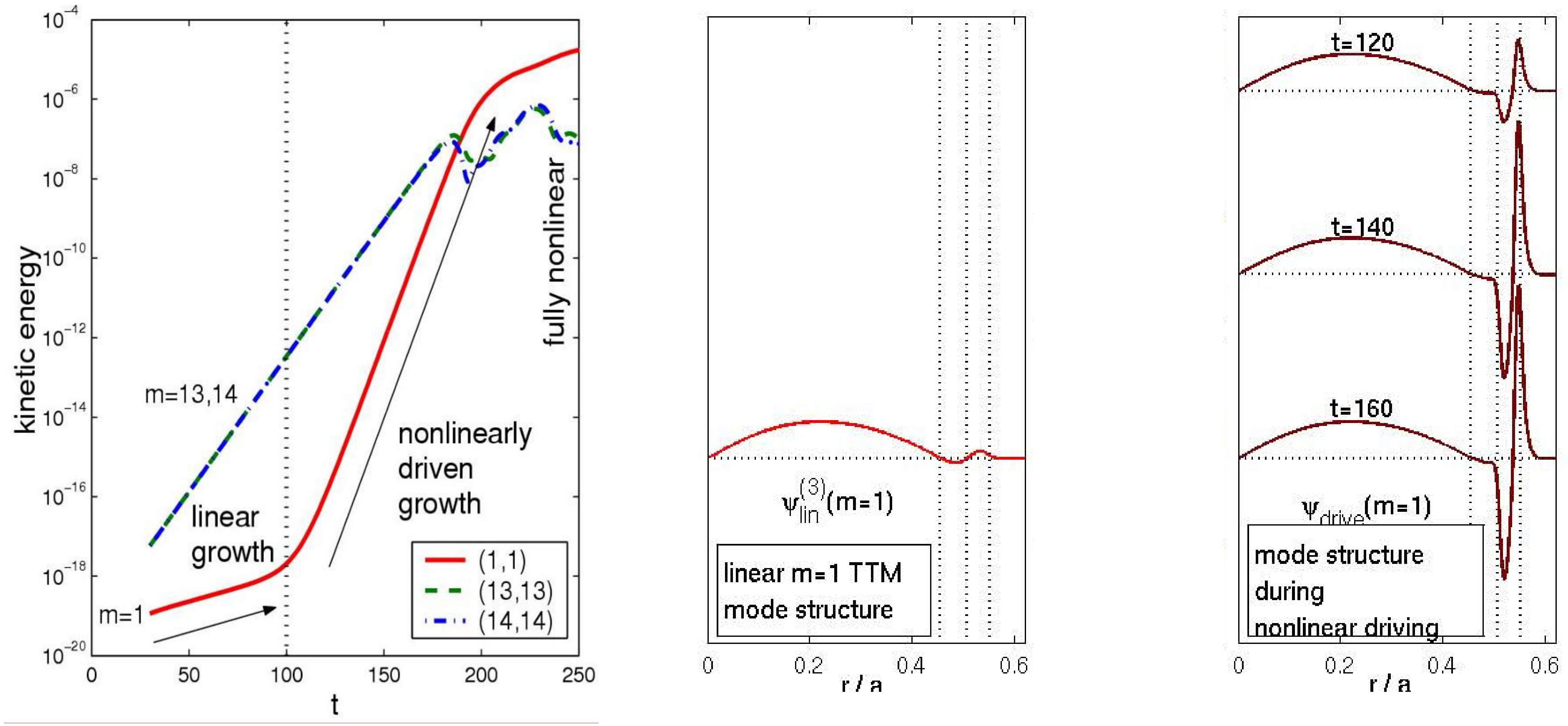




m=1 double kink-tearing mode, m>1 coupled tearing modes

Nonlinear drive of *m=1* mode *(fast trigger)*

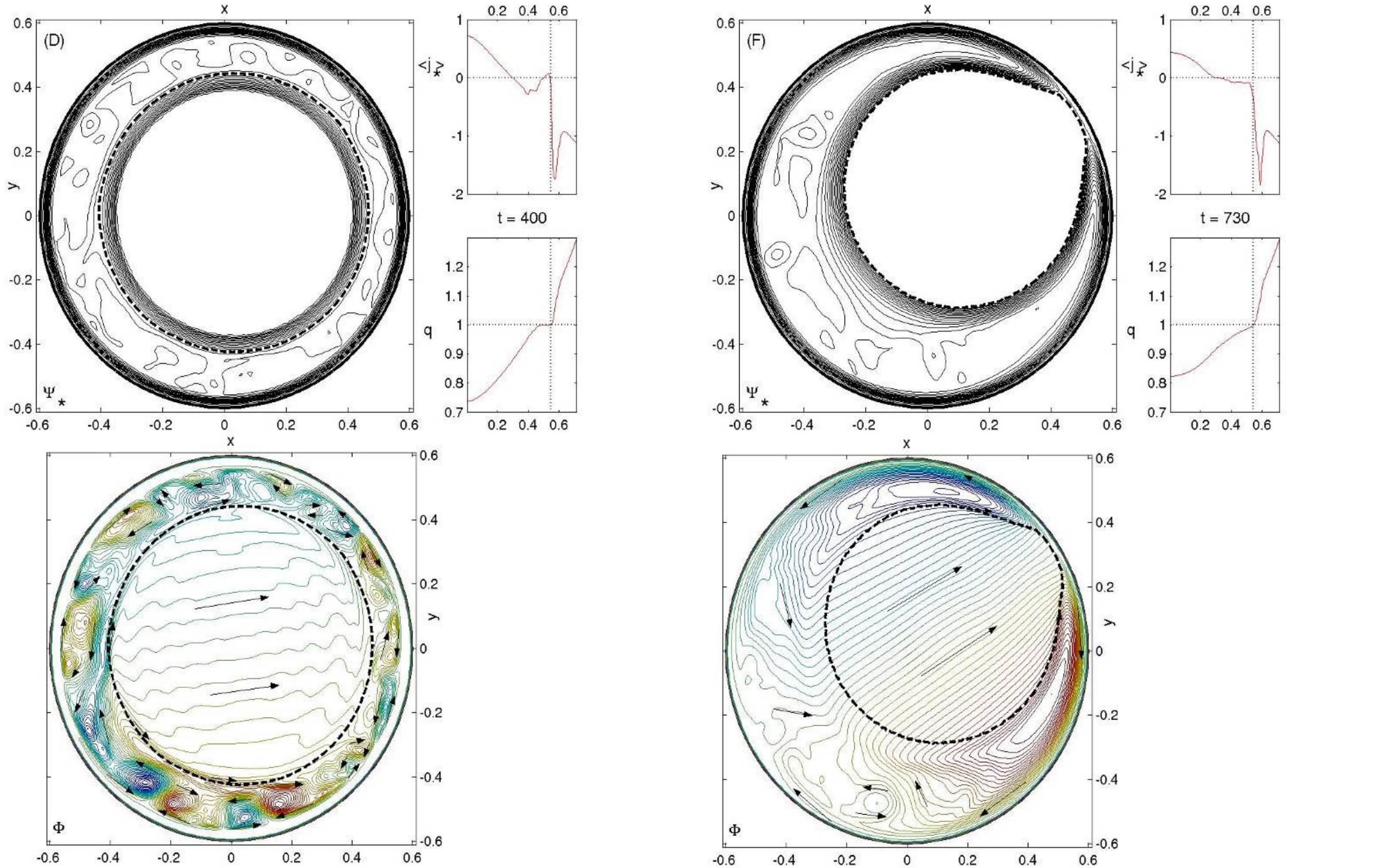
Triple Tearing Modes (TTM) drive resistive kink



- *Growth rate switches from $\gamma_{lin}(m=1)$ to about 2 $\gamma_{lin}(m_{peak})$*
- *Nonlinear drive local, but mode structure remains global*

Sawtooth crash: *Interaction with kink*

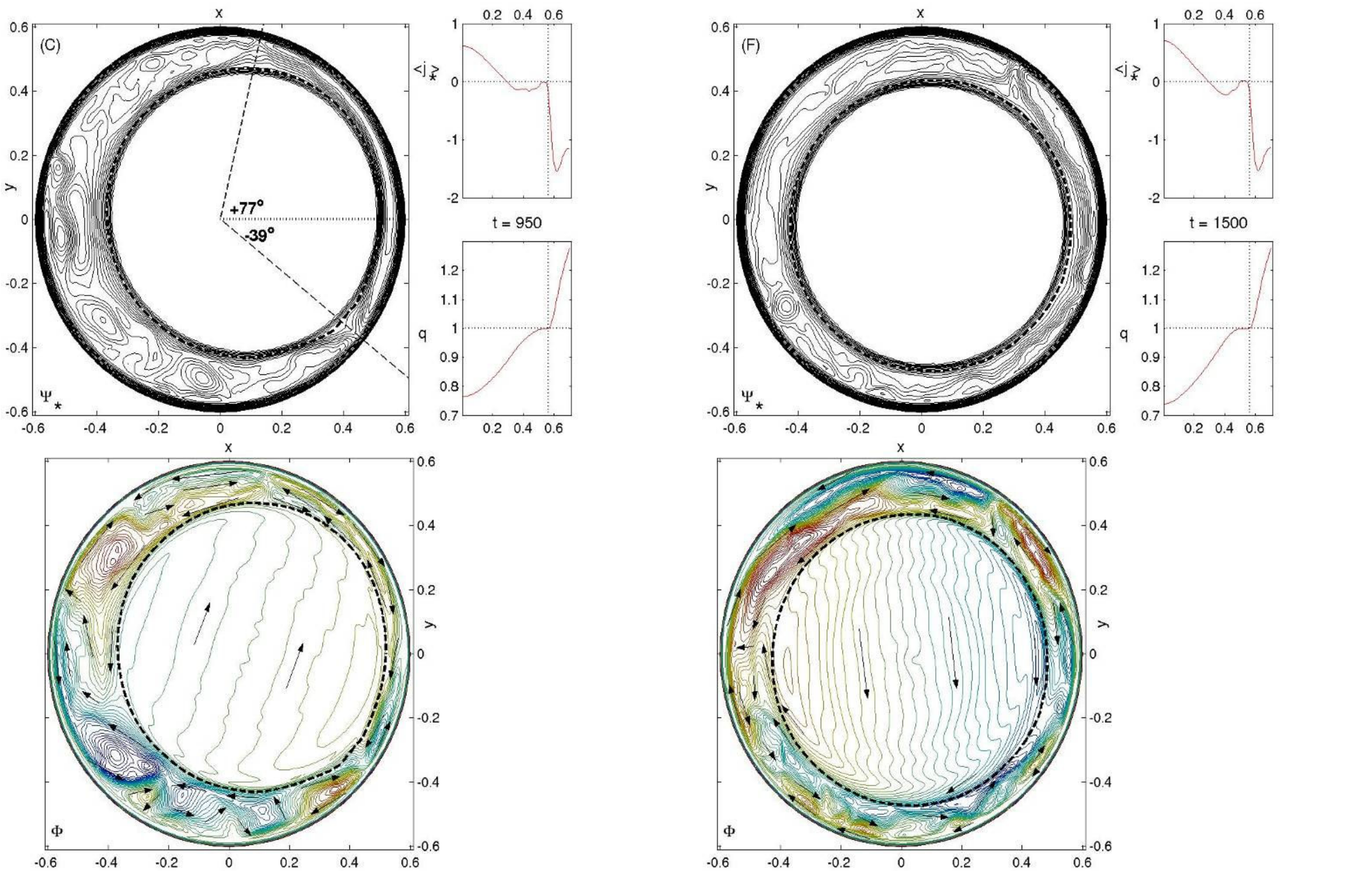
Internal kink surrounded by MHD turbulence



- *Direction of kink flow fluctuates, crash may be delayed*

Sawtooth crash: *Partial reconnection*

Internal kink surrounded by MHD turbulence



- *In some cases, the kink saturates and decays temporarily*

Summary

- Gravitational driven instabilities of fluids: Rayleigh Taylor instability and Rayleigh Bénard instability (convection)
- Similar instability for magnetically confined plasmas: Resistive interchange instability (Resistive g-mode)
- Nonlinear saturation and bifurcation of resistive interchange instability
- Another example of nonlinear evolution and turbulence : Nonlinear evolution of double and triple tearing modes

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S. Benkadda, and M. Wakatani

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theorem

Suppose the range of linear operator L is closed. Then the inhomogeneous equation

$$L\mathbf{x} = \mathbf{a}$$

has a solution if and only if a is orthogonal to all solutions $\mathbf{z}$ of

$$L^{*}\mathbf{z} = \mathbf{0}$$

with L^{*} being the adjoint operator of L .